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RATE OF PRECIPITATION FROM ADIABATICALLY ASCENDING AIR

By J. R. FULKS

[Weather Bureau, Winnemucca, Nev., June 1935]

This paper attempts to present calculated estimates of hourly rates of precipitation in such a manner that the magnitudes of various factors necessary to produce observed rates may be readily visualized. The results are expressed in terms of vertical velocity, temperature, pressure, and thickness of the saturated mass of air. To evaluate all these factors for practical purposes involves serious difficulties; but to any student of meteorology an estimate of their magnitudes, even from a purely theoretical standpoint, is very useful in understanding the processes involved.

Calculations have been made for a layer 100 meters in thickness that has an ascensional rate of 1 meter per second. The range of pressure covered is from 300 mb to 1,000 mb; and of temperature, from $-30°$ C. to $+30°$ C. at 1,000 mb and $-30°$ C. to 0° C. at 300 mb. Instantaneous precipitation of condensed moisture has been assumed (pseudoadiabatic process). The data are plotted on a chart, from which estimates may be made for a layer of any thickness and for any ascensional rate.

Let

ρ=density of dry air in grams per cc.
h=height in centimeters.
Δh=thickness of layer in centimeters.
x=mixing ratio (grams of water vapor per gram of dry air).
T=absolute temperature.
t=time, seconds.
r=rate of precipitation in mm per hour.
ϵ=0.6221, the ratio of densities of water vapor and dry air at same temperature and pressure.
p=atmospheric pressure.
e=saturation vapor pressure.
R=gas constant for 1 gram of dry air.
g=acceleration of gravity.

The rate of precipitation, or more exactly the rate of condensation, represents simply the rate of loss of moisture from the ascending air.

The total weight of moisture in grams in a given mass of air equals the total weight of dry air multiplied by x, the mixing ratio. Therefore, for a thin layer, say 100 meters, in which the density may be considered uniform, the total moisture in grams in a column 1 sq. cm. in cross-section is given by $\rho\Delta h.x$; and the rate of condensation is

$\rho\Delta h\frac{dx}{dt}$ (t in seconds, rate in grams).

This of course is true only when $\rho\Delta h$ (weight of dry air) remains constant, so that the resulting figures are for the particular mass of air under consideration at the instant when its thickness equals the assumed value Δh. The thickness of the layer increases as the density decreases.

Multiplying by 10 to obtain mm of depth instead of grams, and also multiplying by 3,600 to obtain rate per hour, we have

$$r=\rho\Delta h\frac{dx}{dh}\cdot\frac{dh}{dt}36{,}000. \qquad (1)$$

Now

$$x=\frac{\epsilon e}{p-e},$$

and

$$\frac{dx}{dh}=\frac{\epsilon}{(p-e)^2}\left[(p-e)\frac{de}{dT}\cdot\frac{dT}{dh}-e\,\frac{d(p-e)}{dh}\right].$$

Assume that $\frac{d(p-e)}{dh}=-\rho g$, or $\frac{-(p-e)g}{RT\cdot 10^3}$ after dividing by 10^3 to convert pressure from dynes to millibars; this involves an approximation but greatly simplifies the equation: It assumes that $\frac{de}{dh}$ is the density of water vapor times g. It should be understood that this value of $\frac{de}{dh}$ applies *only in this one term*, and not elsewhere. The amount of error produced in the final result by this approximation may be as great as 5 percent in extreme cases; this may appear rather large, but considering the factors which modify true adiabatic conditions it is not serious. The error is greatest for low pressures and high temperatures. Then

$$\frac{dx}{dh}=\frac{\epsilon}{p-e}\left[\frac{de}{dT}\cdot\frac{dT}{dh}+\frac{eg}{RT\cdot 10^3}\right]. \qquad (2)$$

Substituting (2), and $\frac{p-e}{RT}$ for ρ, in equation (1), and making Δh=100 meters or 10^4 cm and $\frac{dh}{dt}$=1 meter or 10^2 cm per second, we obtain

$$r=\frac{3.6\cdot\epsilon\cdot 10^{10}}{RT}\left(\frac{de}{dT}\cdot\frac{dT}{dh}+\frac{eg}{RT\cdot 10^3}\right).$$

Let a be the adiabatic lapse rate in degrees C. per 100 meters, i. e., $\frac{dT}{dh}\cdot 10^4$; and let $b=\frac{de}{dT}$, in mb per degree C.

Then $\frac{de}{dT}\cdot\frac{dT}{dh}=\frac{ab}{10^4}$, and

$$r=\frac{3.6\cdot\epsilon\cdot 10^6}{RT}\left(ab+\frac{10eg}{RT}\right).$$

The quantities a and r are negative, but if their absolute values be taken, the equation becomes

$$r=780\frac{ab}{T}-2{,}666\frac{e}{T^2}, \text{ millimeters per hour.} \qquad (3)$$

It should be remembered that this equation applies only to a 100-meter layer having an ascensional rate of 1 meter per second.

Using this equation, calculations have been made for values of r from 0.1 to 1.0 mm per hour at various temperatures and pressures. Using the values of r thus determined, a chart has been constructed showing the rates of precipitation for various temperatures and pressures. Lines of equal rates may be recognized as those sloping upward to the right. They are shown for each 0.1 mm per hour from 0.1 to 1.0.

For instance, at a pressure of 630 mb and temperature of +1° C. (altitude approximately 4 kilometers) the rate of precipitation from a 100-meter layer having an ascensional rate of 1 meter per second is 0.5 mm or 0.02 inch per hour.

In addition to the hourly rates of precipitation, a few adiabats for saturated air have been drawn on the chart. They are the lines sloping upward to the left.

The chart was constructed by substituting values for r and T in equation (3), solving for a, and then determining the pressure from a table of adiabatic lapse rates; the necessary table of saturated adiabatic lapse rates was calculated from the equation developed by Brunt, *Physical and Dynamical Meteorology*, pages 61–62, but these lapse rates could have been read off with sufficient accuracy from Brunt's diagram. The values of b in equation (3) were found as follows: Vapor pressures over water were taken from the Smithsonian meteorological tables, and over ice from Washburn's table.[1] The equation used in the Smithsonian tables for calculating vapor pressures over water was differentiated, and a table of de/dT calculated from the result. A table of de/dT over ice was calculated from the Clausius-Clapeyron equation (neglecting the specific volume of liquid water); the values of the latent heat of sublimation were first obtained by equating the derivative of the expression given by Washburn for the vapor pressure over ice to the expression given by the Clausius-Clapeyron equation; this process involves less labor than the direct differentiation of Washburn's equation. The saturated adiabatic lines and the height lines were taken from the Neuhoff diagram.

An inspection of the chart will show that the rainfall lines have been extended down to −5° C. These do not coincide with the lines of equal rates of snowfall at the same temperature. The difference is due to two causes: (1) The rate of change of vapor pressure with temperature is greater over ice than over water; this factor tends to increase the rate of precipitation in the snow stage; (2) the heat of sublimation for ice is greater than the heat of condensation for water; this factor tends to decrease the rate of precipitation in the snow stage. It is thus seen that these two factors balance one another to some extent. The actual net result at any given temperature below freezing is that the rate of precipitation at high altitudes is slightly greater in the rain stage, and at low altitudes is slightly greater in the snow stage.

The hail stage has not been considered because we have assumed pseudoadiabatic conditions.

By using the chart it is possible to estimate the rate of precipitation for a layer of any thickness at any given temperature and ascensional rate. The adiabats give an approximation to the lapse rate. As an example we shall take a layer 1 kilometer in thickness having at its base a height of approximately 1 kilometer above the surface. Let the layer have an average vertical velocity of 3 meters per second and a temperature at its base of 10° C. Then, assuming the lapse rate to follow the saturated adiabatic, and reading off the amounts for each 100-meter layer from the top downward: $0.62+0.64+0.65+0.67+0.68+0.69+0.70+0.71+0.72+0.73=6.81$; total rate $=6.81\times 3=20.4$ mm per hour. This is rather heavy rainfall, such as could be expected to occur with strong local convection.

Instead of taking the amount for each 100-meter layer we might estimate the average rate for the layer and multiply by 10 to obtain the rate for the entire layer. In the example just shown this average is about 0.68, or $r=0.68\times 10\times 3=20.4$ mm per hour, the same as before. Converting this example to English units, we have: Thickness of layer=3,300 feet; height of base=about 3,300 feet; temperature at base=50° F.; lapse rate assumed to follow saturated adiabatic; vertical velocity=6.7 miles per hour; rate of precipitation=0.80 inch per hour.

While the ideal conditions assumed may not often obtain in the atmosphere, the results do serve to give some idea of the magnitudes of the various factors necessary to produce observed rates of precipitation.

Suppose that in the previous example the mass of air, instead of moving vertically 3 meters per second, were moving with a horizontal velocity of 25 meters per second upward along a warm front whose slope forces the air to rise uniformly 1 kilometer for each 100 kilometers of horizontal distance. Then the vertical velocity would become ¼ meter per second. Since for 1 meter per second the rate of precipitation was found to be 6.8 mm per hour, the rate in moving up the warm front would become 1.7 mm or 0.07 inch per hour. This is light rain, but continuing steadily as it might along a warm front, would amount to 1.68 inches in 24 hours.

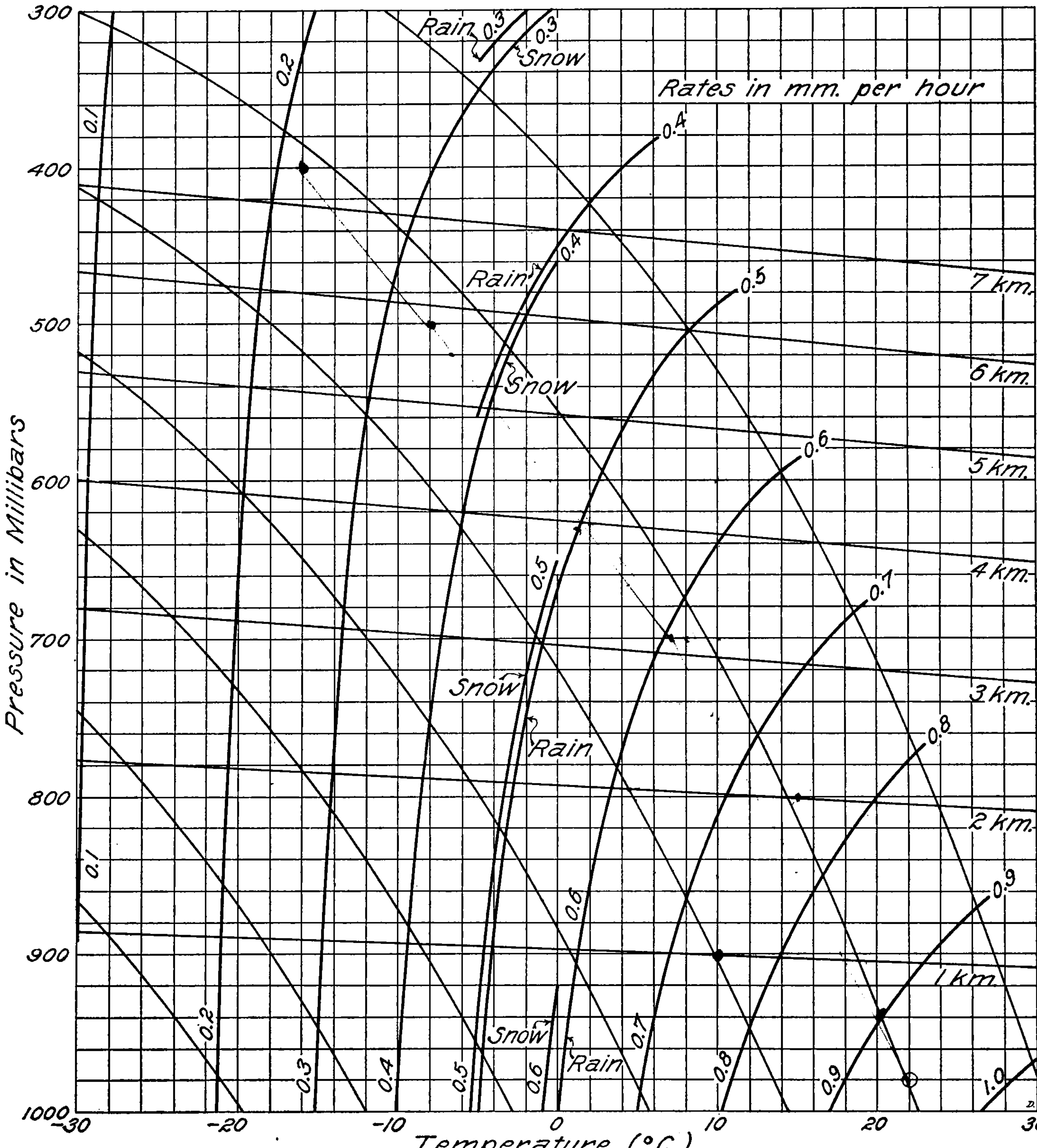
NOTES ON THE FOREGOING PAPER

The relation of rainfall to vertical motion of saturated air has been incidentally considered in various connections by a number of writers, and several more or less rough methods of estimating the possible rates of rainfall have been used;[2] but apparently no explicit formulation of the type developed above by Fulks has previously been given.

In his derivation, Fulks, in addition to assuming ideal pseudoadiabatic conditions, makes two approximations—the change in the thickness of the ascending layer is neglected, and an approximate value is used in one place for de/dh. The following alternative procedure avoids this last assumption: The mass of water vapor in a saturated column of thickness Δh and unit cross-section is $\rho_w\cdot\Delta h$, where ρ_w is the saturation vapor density. Ne-

[1] MON. WEATH. REV., October 1924; see Whipple, MON. WEATH. REV., 1927, p. 131, and Harrison, MON. WEATH. REV., 1934, p. 247.

[2] See e. g., Exner, Sitz Wien. (IIa), Bd. 112, pp. 356–358, 1903; and Dynamische Meteorologie, 2te aufl., pp. 81–82. Scherhag, Ann. d. Hydrog., 63, 36, 1935; Brunt and Douglas, Mem. Roy. Met. Soc., No. 22.



Rates of precipitation from adiabatically ascending air for a 100-meter layer with a vertical velocity of 1 meter per second.

glecting the change in Δh with ascent, which is permissible except for very high ascensional rates, the rate of loss of water during adiabatic ascent is

$$\Delta h\frac{d\rho_w}{dt}=\Delta h\frac{d\rho_w}{dT}\frac{dT}{dh}\frac{dh}{dt};$$

and with the substitution $\rho_w=e/R_wT$, in which $R_w=1.608\ R$ is the characteristic constant for water vapor and e the saturation vapor pressure, we find for a layer 100 meters thick with an ascensional rate of 1 m/sec,

$$r=780\left(\frac{ab}{T}-\frac{ae}{T^2}\right)\text{mm/hr}$$

where a is the lapse rate in °C/100 m, and b the value of de/dT in mb/°C, and in which the mean value of e (in mb) through Δh may be used. Since the second term is very small, this result does not differ appreciably from that obtained by Fulks.—*H. R. Byers.*

In the first of a series of papers by Y. Takahasi, now appearing (in the Japanese language) in the Journal of the Meteorological Society of Japan (vol. 13, pp. 453–455, 1935), the instantaneous rate of condensation at a point in adiabatically ascending saturated air is calculated, and tabulated for various pressures and temperatures, from the equation of continuity; the tabulated values, when multiplied by 36×10^9 to convert to the units employed by Fulks, are in close agreement with the values read from the above chart.

If the slope of a warm front is not uniform as assumed in the example given by Fulks, the rate of precipitation will not remain constant as the warm air moves upward over the colder air; and Ångström has pointed out [3] that the variations in the intensity of precipitation during the passage of a barometric formation may provide some indication of the structure of the fronts that are involved. He shows that if the slope of the surface of discontinuity be constant, the intensity of rainfall will be nearly uniform, but will increase slightly as the formation passes; observed intensity curves during typical front passages, however, show the rate of precipitation to be far from uniform. The shapes which Ångström is thus led to ascribe to fronts are also supported by other types of evidence.

It may be pointed out that the values of de/dT required for the construction of the diagram could have been obtained with equal accuracy and with much less time and labor by simple numerical differentiation of existing tables of vapor pressures (e. g., with Newton's formula) instead of by the method which Fulks describes.—*Edgar W. Woolard.*

[3] Anders Ångström. Die Variation der Niederschlagsintensität bei der Passage von Regengebieten und einige Folgen betreffs der Struktur der Fronten. Met. Zeit., 47: 177–181, 1930.

THE CARIBBEAN HURRICANE OF OCTOBER 19–26, 1935

By W. F. McDONALD

[Weather Bureau, Washington, November 1935]

A tropical cyclone formed between October 17 and 19, 1935, in the western Caribbean Sea, and moved over an unprecedented track which carried the center first northeastward past Jamaica, then in a reverse curve westward near the south coast of Cuba, and finally southwestward to pass inland as a destructive storm over Honduras.

This hurricane was unusual also in another respect; it produced one of the major disasters of West Indian history, causing life losses estimated at perhaps as many as 2,000, without at any time giving evidence of exceptional violence insofar as available wind and barometer observations from ships or land stations along its course are concerned. The losses and damage occurred almost entirely on land areas where the storm winds, impinging on mountainous elevations, produced torrential rains and devastating floods.

As early as the morning of October 17 there was some evidence of a wide-spread but weak cyclonic wind system in the southwestern Caribbean Sea, between Jamaica and Panama. At the same time, a strong anticyclone was centered over the Middle Atlantic States and extended as far eastward as Bermuda and southward to the Florida Straits. Moderate to fresh northerly to easterly gales were reported from October 16 to 19 by ships in several localities northward from the West Indies.

The persistent southward drift of cooler air of continental origin, as high-pressure systems continued to dominate the western Atlantic from October 17 to 22, seems to have been a contributing influence in the further development of the weak cyclone over the western Caribbean, and almost certainly determined the unusual loop backward from the normal course when the center reached the southeast coast of Cuba. The synoptic situation over the North Atlantic on October 18 is shown on chart IX.

The development of this storm first became quite evident on the afternoon of October 19, when the American steamer *Forbes Hauptmann* experienced a south-southwest gale of force 9, with barometer 29.64 inches, near 13° N., 79° W. This report was received by mail and not by radio, and it was not until the next day that ships' radio reports revealed the increased intensity of the storm. The first of these observations was received from the U. S. S. *Chaumont*, on the morning of the 20th, then near 15° N., 77° W., whence she reported south-southeast wind of force 7, and barometer reading 29.68 inches. Twelve hours later the northeastward direction of progression of the disturbance had been determined, and the first advisory warnings of the developing hurricane were issued by the forecast center at Jacksonville, Fla.

The storm moved northeastward as forecast, and the center passed close to Navassa Island during the afternoon of October 21; but the path was even then beginning to deviate northward, and soon thereafter took a more northwesterly direction that brought the center to the coast of Cuba near Santiago, on the early morning of October 22.

Torrential rains over extreme southwestern Haiti attended the storm's passage, and press reports indicated a disastrous total of deaths, the actual number being uncertain but more than 1,000 and possibly as many as 2,000. There was much damage to crops and property in Jamaica, the estimates of monetary losses exceeding $2,000,000. An unidentified schooner and its entire crew were lost off Port Antonio, on the northeast coast, but no other report of deaths from this hurricane has been received from Jamaica.

There was considerable damage in the vicinity of Santiago, Cuba, as the cyclone moved into that region, and press reports indicate that four lives were lost there. The wind exceeded 70 miles per hour at Santiago, as measured by an anemometer on a Pan-American Airways hangar which was blown down after that velocity